Windborne debris risk analysis - Part I. Introduction and methodology

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Abstract. Windborne debris is a major cause of structural damage during severe windstorms and hurricanes owing to its direct impact on building envelopes as well as to the 'chain reaction' failure mechanism it induces by interacting with wind pressure damage. Estimation of debris risk is an important component in evaluating wind damage risk to residential developments. A debris risk model developed by the authors enables one to analytically aggregate damage threats to a building from different types of debris originating from neighboring buildings. This model is extended herein to a general debris risk analysis methodology that is then incorporated into a vulnerability model accounting for the temporal evolution of the interaction between pressure damage and debris damage during storm passage. The current paper (Part I) introduces the debris risk analysis methodology, establishing the mathematical modeling framework. Stochastic models are proposed to estimate the probability distributions of debris trajectory parameters used in the method. It is shown that model statistics can be estimated from available information from wind-tunnel experiments and post-damage surveys. The incorporation of the methodology into vulnerability modeling is described in Part II.

Keywords: windborne debris; risk analysis; probabilistic modeling; hurricane.

1. Introduction

It is widely recognized that windborne debris may cause considerable damage and destruction during severe wind events. In hurricanes, sustained strong winds, changing direction relatively slowly, often lead to partial or complete structural failures that launch building-material debris objects into the wind field and enable them to accelerate to high speeds. Fast-flying debris may cause loss of human life and significant damage to building surfaces. When the building envelopes are penetrated, in addition to the wind and rain damage to building contents, internal pressurization increases the net loading in suction zones, possibly causing failure of roofing and wall cladding, generating new debris and thus starting a 'chain' of failures. According to the damage survey reports of Hurricane Alicia (Texas 1983), Hurricane Hugo (Carolina 1989) and Hurricane Andrew (Florida 1992), windborne debris was a major cause of property damage and a significant contributor to total economic loss (Minor 2005).

Surveys of windborne debris after severe hurricanes in the United States (e.g., Twisdale, *et al.*

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1996) show that a large portion of damaging debris in urban areas is generated from roof cladding and structural components of low-rise buildings, such as roof gravel, tiles, shingles, sheathings, and timber members. These items of debris may be categorized into three generic types: compact (e.g., cubes and spheres), plate-like (e.g., plywood, roof tiles, shingles, and sheathings), and rod-like (e.g., '2 by 4' timbers), as defined by Wills, *et al*. (2002). This classification facilitates development of engineering models for the study of debris aerodynamics. Compact-type objects have little aerodynamic lift force and are likely to fall to the ground under the gravity force. Plate and rod types, on the other hand, develop lift force and rotational moment, which may keep the body in the air for a longer time and allow it to travel farther and gain greater horizontal speed.

The debris problem may be summarized as one of generation, flight, and impact. Debris generation relates to debris restraining conditions and structural damage (for structural-component debris). Debris impact requires characterization of structural resistance. Debris flight trajectories involve the aerodynamics and mechanics of flying objects, and are generally described by flight time, displacement, and velocity. In hurricanes, most debris is generated from elevated sources, such as roofs, and differs from that in tornados where faster wind speeds, combined with a vertical component, tend to pick up heavier debris items from the ground. Also, the principal quantities of interest in the debris object's trajectory in hurricane wind conditions are its horizontal velocity and displacement. Horizontal velocity, together with the mass of the debris, determines the impact strength (which may be quantified by momentum or kinetic energy) when the debris object hits a downwind building. Horizontal displacement determines which downwind building the debris may hit. Vertical displacement and velocity are of secondary importance under hurricane wind conditions, but need to be considered, perhaps through total flight time, to determine whether or not a debris object strikes the ground before impacting a building (Lin, *et al*. 2007).

Tachikawa (1983 and 1988) first conducted experiments on plate and prism debris trajectories in a boundary-layer wind tunnel to support numerical simulation studies, and revealed the existence of flight modes occurring as a function of the initial angle of attack. Tachikawa (1983) also established non-dimensional equations of debris motion and introduced a non-dimensional parameter describing the flight behavior based on both flow and debris characteristics. It has been suggested that this parameter be known as the 'Tachikawa Number' (Holmes, *et al*. 2006). Recently, Lin, *et al*. (2006 and 2007) conducted extensive wind-tunnel experiments to study the flight trajectories of the three generic debris types. They developed, for each debris type, non-dimensional empirical expressions describing debris horizontal speed as a function of horizontal flight distance and horizontal flight distance as a function of flight time, given the value of the Tachikawa Number. Visscher and Kopp (2007) studied trajectories of roof sheathing panels generated from a model house in a wind tunnel, and found that the variability of the trajectory and its sensitivity to particular flight conditions were greater than those observed from earlier experiments in which debris models were launched under well-defined initial support conditions (e.g., Tachikawa 1983; Lin, *et al*. 2006, 2007).

Twisdale, *et al*. (1996) numerically simulated debris flight trajectories using a random orientation 6-degree model (RO 6-D) which considered the motion in three dimensions under drag, lift and side forces; the simulation results were compared with post-damage survey data from hurricane events. Holmes (2004), Holmes, *et al*. (2006), and Lin, *et al*. (2007) described the numerical simulation of two-dimensional motions of various debris types under drag, lift, and moment forces, using wind-tunnel-measured (quasi-steady) force coefficients; Tachikawa's (1983) non-dimensional form was used and the results were validated against wind-tunnel-observed trajectories. Baker (2007) proposed an alternative non-dimensional scheme to that of Tachikawa (1983) and studied numerical solutions of the equations of motion. Richards, *et al*. (2008) performed numerical simulations of three-dimensional motion of plate- and rod-type debris. They found that, released from different initial angles of attack, the positions of a roofing sheet were distributed nearly circularly on a vertical plan at a downstream distance from the release point, which agrees with Tachikawa's (1988) wind-tunnel observations.

There are very few published articles on debris-damage risk assessment. Twisdale, *et al*. (1996) formulated a probabilistic model to estimate the mean debris-damage risk in a residential area, assuming that the impact parameters (e.g., number of impacts, momentum or energy at impact) are identically distributed for all types of debris and for all houses in the study area; also, their debris risk model assumes that the total number of debris impacts on a building has a Poisson distribution. Rather than directly assigning a Poisson distribution to the number of debris impacts, Lin and Vanmarcke (2008) showed that if the number of objects of each type of debris generated from each building in an area can be assumed to have a Poisson distribution, the total number of over-threshold impacts (with impact momentum greater than the resistance) on each building in the area obeys a Poisson distribution. The stochastic processes of debris generation, flight, and impact are thus linked together in this model, enabling one to analytically aggregate damage threats from each type of debris originating from each house in an area, thereby avoiding the common implied assumption of uniformity of risk among houses.

The Twisdale, *et al*. (1996) debris risk model has been applied to vulnerability analysis for residential buildings. They combined numerical models of the hurricane wind field and debris generation, flight, and impact to estimate the probabilistic parameters in their debris risk model. Reliability curves for typical residential "subdivisions" were then produced, and these form the basis of the ASTM recommendations for debris-impact risk analysis (ASTM E1886-05 2005). Also, a simplified approach, based on explicit simulations for typical scenarios, was adopted in the FEMA HAZUS-MH Hurricane Model (Vickery, *et al*. 2006) for hurricane damage and loss estimation. The engineering component of the Florida Public Hurricane Loss Projection (FPHLP) model (Gurley, *et al*. 2005) used another debris risk model, based on the (cumulative) exponential distribution, for structural vulnerability analysis. The expression for the debris damage probability in this model appears similar to that of Twisdale, *et al*. (1996) and of Lin and Vanmarcke (2008), as the exponential distribution and Poisson process are inherently connected. However, estimation of the parameters in this model was greatly simplified, as neither empirical data nor numerical simulations were effectively incorporated.

The current paper, extending the Lin and Vanmarcke (2008) debris risk model, seeks to develop an advanced debris risk analysis methodology that can be conveniently applied to structural vulnerability and reliability analysis. Stochastic models are proposed to estimate the probability distributions of debris trajectory parameters, based on wind-tunnel experimental data from Lin, *et al*. (2006, 2007) and post-damage survey data from Twisdale, *et al*. (1996). Simple numerical examples are given throughout to motivate and illustrate the development of the methodology. This debris risk analysis method, sufficiently flexible and efficient to be applied to general site- and storm-specific analysis, is applied to structural vulnerability assessment for residential neighborhoods in the Part II companion paper (Lin, *et al*. 2010).

2. Windborne debris risk model

The Lin and Vanmarcke (2008) debris model applies the properties of Poisson random measures (see, e.g., Çinlar 2009) to predict the impact damage to a residential area due to debris generated from building sources under hurricane wind conditions. This model estimates the probability of debris damage to building vulnerable areas, without considering the damage to specific vulnerable components of the buildings. In this section, this model is briefly reviewed and then extended to make it more general and applicable to structural reliability and vulnerability analysis, especially the component-based option, in which the risk of debris damage to each window or glass door of a building may need to be quantified (see Part II).

2.1. Review of Lin and Vanmarcke (2008) debris model

The debris risk model of Lin and Vanmarcke (2008) is used to assess the debris damage risk to a group of buildings constituting, for instance, a residential development. First, a relatively isolated residential region is defined such that it is not likely to interact with the outside in terms of debris damage. The relative locations of the residences in the region are fixed. Denote the residences by integers 1, 2, ..., I, where I is their total number within the study area. Every house can generate debris and has a probability of being hit by debris generated from any house in the region, depending on the wind conditions. Suppose there are S types of debris potentially generated from each house, and denote the types by $s = 1, 2, ..., S$. The common debris types that have been observed in hurricane damage surveys are roof covers, roof sheathings, and '2 × 4' timbers ($S = 3$). The subscript i ($i = 1, 2, ..., I$) is used to identify the properties of a house seen as a debris source, and j ($j = 1, 2, ..., I$) when it is considered as an impact target. A_j denotes the area occupied by house j. Denote the impact resistance of the vulnerable area of house j by ζ_j, a threshold expressed in terms of horizontal impact momentum.

Under a particular hurricane wind condition, the number of items of type s debris generated from house i is a random variable, assumed to have a Poisson distribution with mean value $\lambda_{s,i}$. The Poisson assumption is reasonable, as the generation of debris items may be modeled as Bernoulli (binary) trials. Because the generation of a debris item ("success") is an rare event, considering the large number of potential debris sources, the number of generated debris objects is approximately a Poisson random variable (according to the Poisson paradigm; see, e.g., Ross 2007). Define the landing positions of these debris items as independent and identically distributed random variables (taking values on $R \times R$) with $\mu_{s,i}(dx)$ as their common probability distribution, and denote by $\phi_{s,i}(dz|x)$ the conditional distribution of the horizontal impact momentum of the debris item (on R^+), given its landing position.

Then, based on the Poisson random measure theory, it can be shown that the number of debris impacts on house j, from type s debris generated from house i, has a Poisson distribution with mean

$$v_{s,i}(A_j) = \lambda_{s,i}\int_{A_j}\mu_{s,i}(dx) \approx v_{s,i}(j) = \lambda_{s,i}\mu_{s,i}(j)A_j, \tag{1}$$

where $\mu_{s,i}(j)$ denotes $\mu_{s,i}(dx)$ evaluated at the location of the center point of house j. This approximation, adopted to simplify calculations, assumes that the value of $\mu_{s,i}(dx)$ within the area of house j does not vary much.

Moreover, the number of the debris hits on house j having horizontal impact momentum greater than the impact resistance of the house (over-threshold impacts) obeys a Poisson distribution with mean

$$\alpha_{s,i}(A_j) = \lambda_{s,i}\int_{A_j}\mu_{s,i}(dx)\int_{\zeta_j}^{\infty}\phi_{s,i}(dz|x) \approx \alpha_{s,i}(j) = \nu_{s,i}(j)\Phi_{s,i}(Z>\zeta_j|j), \tag{2}$$

where the approximation results, again, from assuming that $\mu_{s,i}(dx)$ and $\phi_{s,i}(dy|x)$ do not change within the area of a house and equal the values at the center point; $\Phi_{s,i}(Z>\zeta_j|j)$ is the probability of the horizontal impact momentum (Z) of a debris object exceeding the impact resistance of the house (ζ_j), or the complementary cumulative distribution function (CCDF) of Z evaluated at ζ_j.

Considering all types of debris generated from all houses in the study area and assuming independent behavior (conditioned on the specific wind condition), the total number of over-threshold debris impacts on a house j obeys a Poisson distribution with mean

$$\alpha(A_j) = \sum_{s=1}^{S}\sum_{i=1}^{I}\alpha_{s,i}(A_j) = \alpha(j) \approx \sum_{s=1}^{S}\sum_{i=1}^{I}\alpha_{s,i}(j). \tag{3}$$

The probability $P(j,N)$ that house j experiences a total of N over-threshold hits is:

$$P(j,N) = \frac{e^{-\alpha(A_j)}[\alpha(A_j)]^N}{N!} \approx \frac{e^{-\alpha(j)}[\alpha(j)]^N}{N!}. \tag{4}$$

Finally, the probability of debris damage, implying at least one over-threshold debris item impacting the vulnerable area, to house j, denoted by $P_D(j)$, is:

$$P_D(j) = \sum_{n=1}^{\infty}\sum_{N=n}^{\infty}P(j,n|N)P(j,N), \tag{5}$$

in which $P(j,n|N)$ is the conditional probability of n impacts on the vulnerable area of house j, given that there are N impacts on house j.

If we assume that the wall vulnerable fraction (the ratio of the vulnerable area on a wall to the area of the wall and the projected roof above it) is approximately the same for all sides of the building and equals the building vulnerable fraction (the ratio of the entire vulnerable area to the area of the building envelope), and that debris impacts on the building walls are uniformly distributed, then $P(j,n|N)$ is approximately a binomial distribution with a "success" probability q_j equal to the building vulnerable fraction,

$$P(j,n|N) = \binom{N}{n}q_j^{\,n}(1-q_j)^{N-n}, \tag{6}$$

and the probability of debris damage to house j becomes:

$$P_D(j) = 1-e^{-q_j\alpha(A_j)} \approx 1-e^{-q_j\alpha(j)}. \tag{7}$$

2.2. Extended debris risk analysis model

Eq. (7) expresses the probability of debris damage at some location within the vulnerable area of a building. The following extension of the model (presented in the approximate form only) enables the method to be used to estimate the debris damage risk for any (specific) vulnerable components on a building envelope.

Given that the number of impacts on house j, from type s debris generated from house i, has a Poisson distribution with mean $\nu_{s,i}(j)$ (given by Eq. (1)), the number of impacts on a defined part of the house (e.g., a window or a door), denoted here by j_*, also has a Poisson distribution, with mean

$$\nu_{s,i}(j_*) = p_{s,i}(j_*|j)\,\nu_{s,i}(j), \tag{8}$$

where $p_{s,i}(j_*|j)$ denotes the probability of a hit involving part j_*, given it occurs somewhere on house j. It then follows that the number of the over-threshold impacts on j_* obeys a Poisson distribution with mean

$$\alpha_{s,i}(j_*) = \nu_{s,i}(j_*)\Phi_{s,i}(Z > \zeta_{j_*}|j_*), \tag{9}$$

in which $\Phi_{s,i}(Z > \zeta_{j_*}|j_*)$ is the CCDF of the horizontal impact momentum of a debris object evaluated at the threshold ζ_{j_*}, the impact resistance of j_*.

Furthermore, the total number of over-threshold hits on j_*, from all types of debris generated from all houses, is also Poisson-distributed, with mean

$$\alpha(j_*) = \sum_{s=1}^{S}\sum_{i=1}^{I}\alpha_{s,i}(j_*). \tag{10}$$

The probability $P(j_*,n)$ that j_* suffers a total of n over-threshold hits is:

$$P(j_*, n) = \frac{e^{-\alpha(j_*)}[\alpha(j_*)]^n}{n!} \tag{11}$$

The probability that there are no over-threshold impacts on j_* is

$$P(j_*, n = 0) = e^{-\alpha(j_*)}, \tag{12}$$

and the probability of debris damage to j_*, denoted by $P_D(j_*)$, is then

$$P_D(j_*) = 1 - e^{-\alpha(j_*)}. \tag{13}$$

In applications to component-based vulnerability modeling, j_* may represent a window (or a glass door) on house j, and $p_{s,i}(j_*|j)$ is then the probability that an impact happens somewhere on the window j_* (given it occurs somewhere on house j). $p_{s,i}(j_*|j)$ will depend on the relative locations and orientations of the source and target houses, as well as on the size and location of the specific window (see Part II). The debris damage risk to each window (or glass door) of a house can then be estimated. It is easy to show that Eq. (13) reduces to Eq. (7) in case j_* is defined as the (entire) vulnerable area of a building and $p_{s,i}(j_*|j)$ is estimated to be the building vulnerable fraction.

2.3. Reliability analysis

The debris risk analysis methodology can be applied to reliability-based design, in which levels of window protection against debris damage need to satisfy reliability and/or performance goals. As a simple example, denote the reliability of building j with respect to debris damage by $R(j) = 1 - P_D(j)$. Eq. (7) may be written as:

$$\alpha(j) = -\frac{1}{q_j} ln[R(j)] . \tag{14}$$

If there is only one type of debris generated from one house, substituting Eqs. (1) and (2) into Eq. (14), we obtain:

$$\Phi(Z > \zeta_j | j) = -\frac{1}{\lambda q_j \mu(j) A_j} ln[R(j)] , \tag{15}$$

enabling the required impact resistance of the window protection, ζ_j, to be estimated for any given reliability goal. A similar analysis can be carried out if the reliability of a particular window is considered, making use of Eq. (13) and the conditional probability of debris impacting the window $p(j_* | j)$.

It should be noted, however, that the required impact resistance of the window protection can be estimated by analytical derivation only when just one type of debris generated from one house is considered. In reality, there are usually several types of debris generated from many houses in a residential development, and thus the debris risk has to be estimated by aggregating the damage threats from all these debris sources. Also, if the extended model is used to estimate the reliability of the entire window system of a building (because, for instance, the wall vulnerable fractions vary significantly for different sides of the building), the probability of debris damage to the window area of the building will depend on the sum of the mean number of over-threshold impacts on each window. As a consequence, the required level of window protection cannot be estimated analytically, and some iteration will be required. A design level (ζ) has to be assumed first in order to estimate the probability of window damage (using Eq. (7) or Eq. (13)). If the reliability goal is not satisfied, the design level has to be increased and the damage probability estimated again, and so on, until the reliability goal is met. The number, size, and location of windows (related to q or p) might also be changed in design process, in order to achieve the reliability goal.

3. Stochastic debris flight trajectory

The debris risk model developed in Section 2 makes use of four probabilistic parameters or distributions for each type of debris generated from each building in an area: the mean number of debris objects generated (λ), the probability distribution of debris landing positions on the horizontal plan (μ), the conditional probability of debris impacting a vulnerable building component (p), and the conditional probability distribution of horizontal impact momentum (ϕ). The parameter λ may be estimated from a component-based pressure damage model, while p is estimated based on the dimensional characteristics of the houses; these are discussed in detail in Part II. The aim of the stochastic debris trajectory model developed in this section is to obtain the other two probabilistic quantities, μ and ϕ. Since observation of debris trajectories in real storms is generally not available and post-damage survey data is very limited, the estimation of the probability distributions of debris trajectory parameters relies mainly on experimental and numerical simulations, and, given the complexity of the problem in real situations, intuition and common sense.

Tachikawa (1983) established non-dimensional (scaled) equations of debris motion in two dimensions, based on Newton's second law. The non-dimensional horizontal flight speed, horizontal flight distance, and flight time in the equations are:

$$\tilde{u} = \frac{u}{W}, \quad \tilde{x} = \frac{gx}{W^2}, \quad \tilde{t} = \frac{gt}{W}, \tag{16}$$

where W is the wind speed, u is debris horizontal speed, x is horizontal flight distance, t is flight time, and g is the acceleration of gravity. Tachikawa (1983) also identified, based on the equations, the dimensionless Tachikawa Number K (Holmes, *et al*. 2006), measuring the relationship between the aerodynamic and gravity forces:

$$K = \frac{\rho_a W^2 A_d}{2M_d g} = \frac{\rho_a W^2}{2gh_d\rho_d}, \tag{17}$$

in which A_d is debris area, M_d is debris mass, h_d is debris thickness, ρ_d is debris density, and ρ_a is air density.

Lin, *et al*. (2006, 2007) extensively tested trajectories of the three generic types of debris (i.e., compact, plate-like and rod-like debris) in the Texas Tech University wind tunnel, under a wide range of wind speeds and experimental settings. They observed that although the trajectories of a certain type of debris greatly varied in the vertical direction, their horizontal components exhibited definite patterns, dependent mainly on the Tachikawa Number K. They also proposed expressions, in dimensionless form, to describe debris horizontal trajectories, based on the empirical data and theoretical reasoning (see Lin, *et al*. 2007). The horizontal speed is expressed as a function of flight distance as follows:

$$\tilde{u} \approx 1 - e^{-\sqrt{2CK\tilde{x}}}, \tag{18}$$

where C, a constant that depends on the shape of the debris, is regarded as an aerodynamic coefficient for debris horizontal trajectory. The horizontal flight distance is expressed as a function of flight time as:

$$K\tilde{x} \approx \frac{1}{2}C(K\tilde{t})^2 + c_1(K\tilde{t})^3 + c_2(K\tilde{t})^4 + c_3(K\tilde{t})^5, \tag{19}$$

where the constant coefficient C is the same as in Eq. (18), and the coefficients c_1, c_2, and c_3 also depend on the shape of the debris. Their numerical values were estimated by least-square regression analysis of a large amount of empirical data for plate-type (Lin, *et al*. 2006), compact and rod-type (Lin, *et al*. 2007) debris, respectively. The empirical expressions satisfactorily match numerical simulations (see Holmes, *et al*. 2006; Lin, *et al*. 2007).

It should be noted that these empirical expressions of debris trajectory parameters were developed based on experimental data for a limited range of debris models and wind speeds realizable in the wind tunnel. In order to obtain a good fit to the data, high polynomial orders appear in Eq. (19), which, for this reason, should not be used for extrapolation. For instance, the range of $K\tilde{t}$ in the experiments was about [0, 6.5] for plate-like debris (Lin, *et al*., 2006). In case $K\tilde{t} > 6.5$, the flight distance up to $K\tilde{t} = 6.5$ can be estimated first from Eq. (19), and the added flight distance may be obtained by assuming that the debris flies with a constant speed approaching the wind speed (because, beyond the available estimation range, most debris items should have achieved a high flight speed, close to the wind speed).

The equations of motion of debris trajectories and the above empirical relationships describe the deterministic aspects of debris trajectories in controlled environments. Debris trajectories in nature, however, are much less predictable, due to other effects that are not easily parameterized, such as the irregularity of actual debris shapes, initial support conditions, and turbulence in hurricane winds. In order to estimate the probability distributions of actual debris trajectory parameters, larger data sets, obtainable from numerical simulations, model-scale and full-scale experiments, and storm observations and post-damage surveys, are needed. For instance, the probabilistic character of debris trajectories may be studied by carrying out Monte Carlo simulations, randomizing pertinent variables in each trial. Wind-tunnel experiments on model houses (e.g., Visscher and Kopp 2007) may complement information from experiments on trajectories of items of debris originating from simple supports. Lin, *et al*. (2006) conducted full-scale tests on trajectories of plates, using C-130 aircraft to generate strong winds (see Lin 2005 for details). Such full-scale experiments allow the entire trajectory to be observed until the debris strikes the ground, unlike in a wind tunnel (due to its limited size). New experimental wind generation methods may be applied, such as the Wall-of-Wind hurricane simulation facility (e.g., Leatherman, *et al*. 2007); destructive testing (in which debris is generated from full-scale houses) may produce debris whose trajectories are close to these in real storms. Even real debris trajectories may be captured in the future by installing high-speed video cameras in coastal residential areas along the predicted storm path. Advanced statistical methods may be applied to combine the different sources of data to overcome the limitation of each data source.

In the following subsections, we propose stochastic models of debris trajectory parameters based on available observations. The first-order statistics of these models are estimated mainly from the experimental data of Lin, *et al*. (2006, 2007; Eqs. (18) and (19)), assuming that the controlled wind-tunnel debris trajectories represent the mean of debris trajectories in nature. Second-order statistics are obtained by linking the variation of the debris trajectory to the mean trajectory, in a way that satisfies physical constraints and is consistent with post-damage observations.

3.1. Debris flight speed

We propose modeling the non-dimensional debris horizontal flight speed (and impact speed) $\tilde{u}$ with a Beta distribution, widely used to represent random quantities restricted to the interval [0,1]. Parameterized by two positive shape parameters, denoted by a and b, the probability density function can be unimodal, U-shaped, J-shaped, or uniform. The probability density function (PDF) of the Beta distribution for $\tilde{u}$ is

$$f_{\tilde{u}}(x) = \frac{x^{a-1}(1-x)^{b-1}}{B(a,b)}, \; x \in [0, 1], \tag{20}$$

where $B(\cdot, \cdot)$ denotes the Beta function, $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, expressed in terms of the Gamma function $\Gamma(\cdot)$. The Beta distribution is also uniquely determined by its mean ($m_{\tilde{u}}$) and dispersion ($\eta_{\tilde{u}}$), as the shape parameters can be expressed in terms of the mean and dispersion as follows:

$$a = m_{\tilde{u}}\eta_{\tilde{u}}, \quad b = (1 - m_{\tilde{u}})\eta_{\tilde{u}}. \tag{21}$$

Studies have indicated that, as the debris accelerates towards the wind speed, the non-dimensional debris horizontal speed $\tilde{u}$ approaches a value of 1.0 asymptotically (Lin, *et al*. 2007), so $0 \le \tilde{u} \le 1$, indicating the Beta distribution to be a good model candidate. Moreover, if we set $a > 1$ and $b > 1$ the density function of $\tilde{u}$ will be unimodal, peaking in the interior of [0,1] and decaying to zeros at the range limits, consistent with the physics of debris flight.

We assume the mean of $\tilde{u}$ to be a function of $\tilde{x}$ (the non-dimensional flight distance), as expressed in Eq. (18), namely,

$$m_{\tilde{u}} = 1 - e^{-\sqrt{2CK\tilde{x}}}, \tag{22}$$

where C is obtained from Lin, *et al*. (2006) for plate-like debris and from Lin, *et al*. (2007) for compact or rod-like debris. In order to satisfy the constraints of $a > 1$ and $b > 1$, the dispersion parameter $\eta_{\tilde{u}}$ must be larger than max($1/m_{\tilde{u}}$,$1/(1-m_{\tilde{u}})$), according to Eq. (21). We may assume that

$$\eta_{\tilde{u}} = \max\left(\frac{1}{m_{\tilde{u}}}, \frac{1}{1-m_{\tilde{u}}}\right) + \gamma, \tag{23}$$

with γ being a strictly positive number to be estimated (for each debris type) when new data become available. In this way, the shape parameters a and b in the Beta distribution of the non-dimensional debris horizontal speed (Eq. (20)) can be obtained.

Assuming the mass of a given debris object (M_d) to be a known constant, the probability distribution (ϕ) of debris horizontal momentum ($Z = M_d u$) is also obtained, and the exceedance probability of a debris object's horizontal momentum, when it hits a house (j), can be calculated as follows:

$$\Phi(Z > \zeta_j | j) = \Phi\left(\tilde{u} > \frac{\zeta_j}{WM_d}\middle| j\right) = \int_{\frac{\zeta_j}{WM_d}}^{1} f_{\tilde{u}}(x)dx, \tag{24}$$

where $f_{\tilde{u}}(x)$ is the probability distribution function of $\tilde{u}$. Here, debris flight distance to impact is thought of (and approximated as) the distance between the center points of the source and target buildings. If the risk of debris damage to a vulnerable component (e.g., window j_*) is sought, the flight distance to impact may be estimated as the distance between the center points of the source house and of the vulnerable component when calculating $\Phi(Z > \zeta_{j_*} | j_*)$ (Eq. 9). The distance between the center points of the source and target buildings may also be used as a general approximation for the flight distance to impact, thereby ignoring the effect on the debris horizontal impact speed of the location of the vulnerable component on the target building.

As a numerical example, consider a typical roof tile with area of $A_d = 0.143$ m^2 and weight $M_d = 4.08$ kg. When it travels a distance of 30 m, 60 m, and 90 m, respectively, the means of its non-dimensional horizontal speed ($m_{\tilde{u}}$) are 0.66, 0.79, and 0.84, respectively (from Eq. (22), using $C = 0.91$ for plate-like debris; Lin, *et al*. 2006). The corresponding Beta PDF and CCDF of $\tilde{u}$ (assuming $\gamma = 3$) are given in Fig. 1. Obviously, the farther the tile debris object flies, the higher the non-dimensional horizontal speed it is likely to achieve and the smaller the variation of the speed, consistent with debris flight characteristics. The Beta CCDF describes the probability of $\tilde{u}$

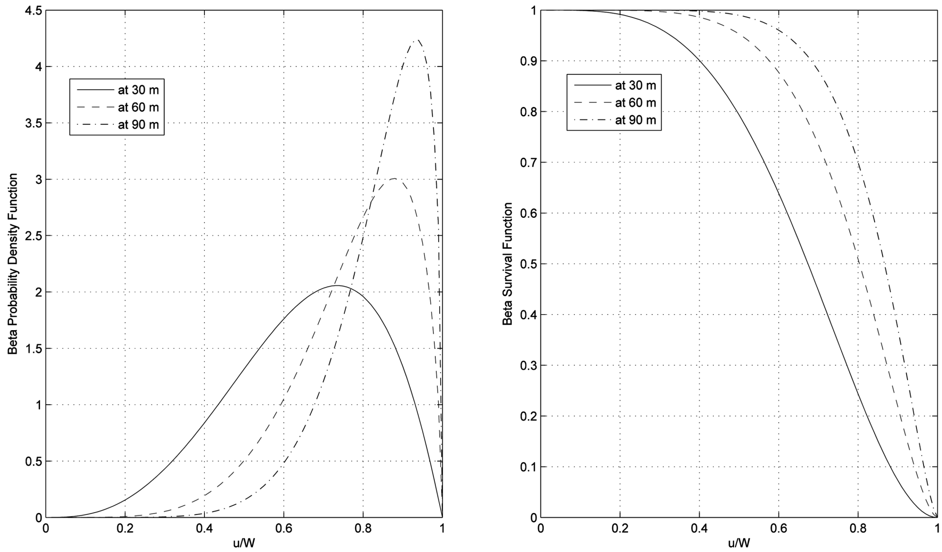

Fig. 1 The Beta probability density function (PDF) and complementary cumulative density function (CCDF) for the non-dimensional horizontal flight speed of a piece of tile debris at different travel distances

exceeding a given value; for instance, the probability of $\tilde{u} > 0.4$ is about 0.90 for 30-m, 0.99 for 60-m, and 1.0 for 90-m flight distance. Suppose the wind speed is 49 m s^{-1} and the impact resistance (in terms of horizontal impact momentum) of the windows (or their protection covers) is 80 kg m s^{-1}; then $\zeta_j / W M_d = 0.4$. The probability of some window being damaged ($\Phi(Z > \zeta_j | j)$; Eq. (24)) is 0.90 if it is hit by the tile debris object from a distance of 30 m; 0.99 if the distance is 60 m; and 1.0 if the distance is 90 m. However, this does not mean that a building is more vulnerable to debris damage if it is farther from the debris source, as the building may never be hit at a far distance. Clearly, the probability of debris impact also has to be counted in when estimating debris damage risk.

3.2. Debris flight displacement and time

We model the probability distribution of debris landing positions (μ) with a two-dimensional Gaussian distribution. This is motivated by observations from wind-tunnel experiments (Tachikawa 1988) and numerical simulations (Richards, *et al.* 2008). Tachikawa (1988) placed a catch-net perpendicular to the direction of the wind at various distances in front of the debris original position and found that debris impact locations were always almost uniformly distributed within circles on the net. Richards, *et al.* (2008) conducted numerical simulations for a full-scale roofing sheet released from different initial angles of attack; the positions of the roofing sheet were distributed nearly circularly on a vertical plan at 50 m downstream from the release point. These observations indicate that the variation of the trajectory is nearly symmetric in both along-wind and across-wind directions. Thus, a two-dimensional Gaussian distribution appears to be a good candidate for modeling the distribution of debris landing positions.

The probability density function of a two-dimensional Gaussian distribution of debris landing position is

$$\mu(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left[-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{x-m_x}{\sigma_x}\right)^2 - 2\rho\frac{(x-m_x)}{\sigma_x}\frac{(y-m_y)}{\sigma_y} + \left(\frac{y-m_y}{\sigma_y}\right)^2\right\}\right], \tag{25}$$

where m_x and σ_x represent the mean and variance of the along-wind displacement, and m_y and σ_y are those of the across-wind displacement; ρ is the correlation coefficient between the displacements in the two directions. These five parameters completely determine the two-dimensional Gaussian distribution.

Denote by T the debris object's travel time until it lands on the ground. We assume the mean of the total along-wind displacement to be a function of T, as expressed in Eq. (19); that is,

$$m_x = \frac{2\rho_d h_d}{\rho_a}\left[\frac{1}{2}C(K\tilde{T})^2 + c_1(K\tilde{T})^3 + c_2(K\tilde{T})^4 + c_3(K\tilde{T})^5\right], \tag{26}$$

where $\tilde{T} = gT/W$ and the values of the (constant) coefficients for plate-like debris are obtained from Lin, *et al*. (2006), and for compact and rod-like debris from Lin, *et al*. (2007), respectively. We also assume the coefficient of variation (COV) of the along-wind displacement, denoted by ς, to be a positive constant for each type of debris, so that the standard deviation of the along-wind displacement is

$$\sigma_x = \varsigma m_x, \tag{27}$$

indicating that the farther a debris object flies the larger the variation of its flight displacement.

We assume the mean of the total across-wind displacement to be zero,

$$m_y = 0. \tag{28}$$

According to the observation of nearly symmetrical variation of the debris trajectory in along-wind and across-wind directions, we assume that the standard deviations of the across-wind and along-wind displacements are equal,

$$\sigma_y = \sigma_x. \tag{29}$$

Also, lacking data about debris trajectories in the across-wind direction, in the current model we assume the across-wind and along-wind displacements to be statistically independent, hence

$$\rho = 0. \tag{30}$$

It has been found from wind-tunnel experiments (e.g., Tachikawa 1983; Lin, *et al*. 2006) and numerical simulations (e.g., Holmes, *et al*. 2006) that the trajectory pattern of plate-like and rod-like debris objects mainly depends on their mode of motion; a debris object may fall to the ground quickly or fly up and stay in the air for much longer time, depending on the initial support configuration and initial angle of attack. Post-damage survey data in Twisdale, *et al*. (1996) also show that, among the similar sheathing debris items originating from the same roof, some landed nearby while others reached much greater distances. Quantitative data on debris flight time, however, is very limited, as debris flight time until landing is often not observed in a wind tunnel (due to its limited size) and is not observable during a real storm without special instruments.

Nevertheless, we may estimate the debris flight time from post-damage survey data on debris transport. For example, post-damage surveys by Twisdale, *et al.* (1996) revealed dimensional characteristics and transport distances of 13 pieces of sheathing and tile debris after Hurricanes Erin and Opal (1995) made landfall in Florida. Assuming debris flight in the along-wind direction, one can estimate the flight time of each debris item using Eq. (19) (with $C = 0.91$, $c_1 = -0.148$, $c_2 = 0.024$, and $c_3 = -0.0014$ for plate-like debris; Lin, *et al.* 2006), given the observed transport distance. The two modes of motion are clearly shown in the data: 5 debris items flew less than 1 s (with a mean of 0.73 s) before falling to the ground, while the other 8 debris items stayed in the air much longer, up to 2.2 s (with a mean of 1.58 s), and reached much greater distances. As a first approximation, based on this limited data set, we assume the flight time of debris (plate-type in this case) to be a discrete random variable taking either a low value (near 0.73 s) or a higher value (near 1.58 s), with probabilities 5/13 and 8/13 (or, roughly, 1/3 and 2/3), respectively. (This is consistent with other observations from damage surveys and full-scale experiments (Lin, *et al.* 2006) indicating that debris items generally fly in the air for about 1 to 3 seconds.) The variation of flight distance within each mode may be quantified in terms of the coefficient of variation of the along-wind displacement (ς), calculated for each sample, by noting the difference between the observed flight distance and the predicted mean value (by Eq. (26) with $T = 0.73$ s or $T = 1.58$ s, depending on the observed mode); $\varsigma \approx 0.35$ is the average value for all of the (13) samples. Fig. 2 shows the comparison between the observed and simulated (using the estimated parameters) flight distances for the 13 debris sample cases. Simulated debris flight distances span relatively large ranges, indicative of the randomness of debris trajectories in nature. The observed debris flight distance falls around the mean of one of the two modes of the simulated flight distances in most cases. This method to estimate the debris flight time is similarly applicable for rod-like debris and compact

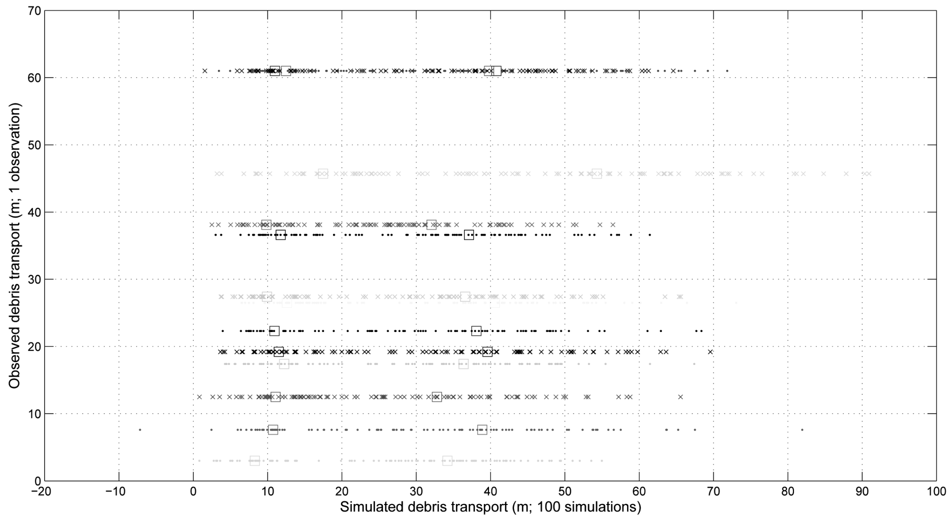

Fig. 2 Comparison between the observed (Twisdale, *et al.* 1996) and simulated transport distances for 13 debris samples. (Hollow square symbols mark the mean of each of the two modes of the simulated flight distances)

debris; the latter may be reduced to a special case with one-mode ("falling-down") debris flight time.

With only very limited data available on the full-scale debris transport, the above method to estimate debris flight time and the variation of flight displacement represents a first attempt to incorporate damage-survey data into debris risk analysis. When sufficient data become available in the future (from experiments, numerical simulations, and post-damage surveys), debris total flight time may presumably be modeled as a continuous random variable with a two-mode mixed Gaussian distribution. Moreover, for each mode the flight time may depend on the value of Tachikawa Number K and the initial height at which the debris object starts to fly; the greater the value of K and the higher the initial location, the longer the expected total time the debris will stay in the air. Such dependence can be modeled by regression methods, linking the mean and COV of each mode to the value of K and the initial height.

Consider again the roof tile involved in the numerical example in Section 3.1. Under the assumption that its trajectory has two modes with mean total flight time (T) of 0.73 s and 1.58 s, respectively, for a wind speed of 49 m s^{-1}, its mean flight distance in the wind direction is estimated as 9.5 m for the first mode and 34.0 m for the second mode (by Eq. (26) for plate-like debris). (This is consistent with the observation of Twisdale, *et al*. (1996) that ten such tiles were transported about 38 m before impacting another building, under a similar wind speed.) Assuming $\varsigma = 0.35$, the value of the probability density function of debris landing at any location can be calculated for each of the two modes (by Eq. (25)). The probability density function of debris landing (μ) is the weighted sum over the two modes, the weights being the probabilities of debris flying according to each mode: 0.38 (5/13) for the first and 0.62 (8/13) for the second mode. The value of μ at a distance in the downwind direction at 30 m, 60 m, and 90 m is 6.52×10^{-4}, 6.62×10^{-5}, and 1.22×10^{-8}, respectively. Considering a house with a plan area (A_j) of 245 m^2, for instance, the probability of it being hit ($\mu(j)A_j$) is 0.16 if it is centered at 30 m, 0.016 if at 60 m, and 0.0 if at 90 m, in the downwind direction from the original location of the tile debris.

Suppose there are, on average, 50 such roof tiles generated from a (source) house ($\lambda = 50$), under the wind speed of 49 m s^{-1}. The mean number of hits of these tiles on the (target) house of 245 m^2 ($\nu(A_j) = \lambda\mu(j)A_j$; Eq. (1)) is about 7.99, 0.81, and 0.0, if it is centered at 30 m, 60 m, and 90 m, respectively, in the downwind direction from the source house. If the impact resistance of the target house is $\zeta_j = 80$ kg m s^{-1}, the mean number of over-threshold hits ($\alpha(j) = \nu(A_j)\Phi(Z > \zeta_j|j)$; Eq. (2)) is 7.20, 0.80, and 0.0, respectively. (The values of $\Phi(Z > \zeta_j|j)$ are obtained from the numerical example in Section 3.1, neglecting the effect of house dimensions in calculating the flight distance to impact.) Suppose the window fraction (q_j) of the target house is 0.15, then the probability of debris damage to the window area of the target building ($P_D(j)$; Eq. (7)) is 0.66, 0.11, and 0.0, when it is centered at 30 m, 60 m, and 90 m, respectively, in the downwind direction from the source house.

4. Conclusions

The Lin and Vanmarcke (2008) debris risk model is extended into a more general debris risk analysis methodology, based on Poisson random measure theory. A new parameter is introduced to describe the chance of debris impact on particular vulnerable components on a building envelope, so the model can be used to estimate debris damage risk for each of the vulnerable components. The application of this debris risk model to reliability analysis is discussed. The method is further applied, in combination with a component-based pressure damage model, to structural vulnerability analysis in the Part II companion paper (Lin, *et al*. 2010).

Stochastic models are proposed for debris trajectory parameters. In a first effort, wind-tunnel experimental data and post-damage survey data are used to estimate model statistics in debris risk analysis. However, available information on debris generation, flight, and damage, especially for the purpose of quantifying probabilistic aspects of the problem, is currently very limited. More debris-related data is needed from experimental and numerical simulations and from observations during or after real storms. Improved statistical methods need to be developed and applied to combine different data sources, in order to improve the accuracy of current debris risk estimation.

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